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LOCAL GEOID AND GRAVITY ANOMALY PREDICTIONS USING POINT MASSES.(U)

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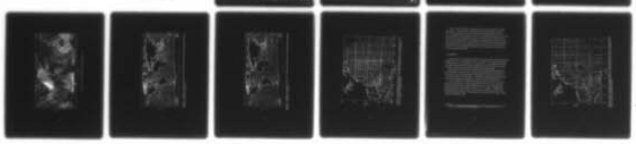

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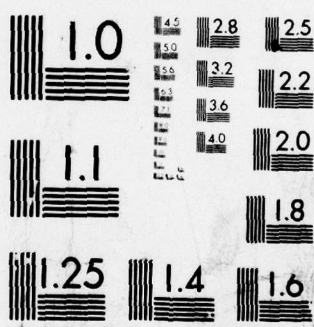

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Local Geoid and Gravity Anomaly Predictions Using Point Masses

GEORGES BLAHA
GEORGE HADGIGEORGE

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This report has been reviewed by the ESD Information Office (OI) and is releasable to the National Technical Information Service (NTIS).

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FOR THE COMMANDER

John Howard

Chief Scientist

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The point mass technique, which supplements a spherical harmonic expansion of the potential, allows short wavelength detail to be added to previously computed long wavelength geoidal features without distorting the latter. Recently, the AFGL short-arc approach to satellite altimetry has been modified to allow point mass adjustments. GEOS-3 altimeter residuals in the North Atlantic region were obtained from a first adjustment in terms of spherical harmonics and state vector parameters. A subsequent point mass adjustment was performed on these residuals to obtain the short wavelength geoid → next page

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and gravity anomalies. Advantages of the point mass model stem from a flexible deployment of the new parameters (point mass magnitudes) in an area of interest that permits important computer savings when processing large amounts of satellite data in a local region.

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1. INTRODUCTION

In order to represent the geoid to a desired resolution using spherical harmonics, the necessary number of potential coefficients is exceedingly large. For example, the resolution corresponding roughly to 1°(about 111 km) requires a set of coefficients complete through degree and order (180, 180) involving almost 33,000 coefficients. To produce a resolution twice as fine, that is, about 0.5°, four times as many coefficients are required. If enormous quantities of data were available all over the globe so that such sets of coefficients would be theoretically solvable in a least squares adjustment, the matrix of normal equations in the 1° case would have dimensions of almost 33,000 × 33,000. The difficulties connected with the actual solution of this system would be formidable. Thus, even to describe the geoid over a relatively small region, such as the North Atlantic, would be extremely wasteful if not impossible.

The use of local parameters called "point masses" has been envisioned as a technique to avoid some problems associated with the spherical harmonics. Point masses have the same meaning as other terms used in the geodetic literature, for example, "buried masses" or "mass concentrations". Point masses are useful because they can give rise to the disturbing potential from which geophysically

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meaningful quantities such as geoid undulations, gravity anomalies, and so on, can be derived.

The disturbing potential added to the normal potential results in the total potential. A method whereby point masses give rise to the disturbing potential has been presented in Reilly and Herbrechtsmeier.[1] Another possibility is separation of the disturbing potential (T) into two parts, $T_{S.H.}$ and $T_{P.M.}$; $T_{S.H.}$ is represented by a spherical harmonic expansion and $T_{P.M.}$ by the point mass model. This is, in fact, the avenue pursued in this report.

The idea of supplementing the spherical harmonic expansion of the earth's potential by point masses is relatively recent. The purpose for introducing the point mass parameters into an adjustment of gravity anomalies, satellite altimetry, and perhaps other quantities as well, is to add fine structure to a geopotential model based on spherical harmonic coefficients. The point mass locations, including their depth, theoretically stipulated with respect to the geoid or a reference ellipsoid are the object of judicial choice, while the point mass magnitudes are subjected to an adjustment in a generally over-determined system.

A first adjustment in terms of spherical harmonic potential coefficients accommodates the available geoid undulations, obtained from satellite altimetry, as well as mean gravity anomaly data. A subsequent localized adjustment is based on the altimetry residuals computed in a region of interest from the first adjustment; these residuals are accommodated in an additional data fit through the use of a suitable number of new parameters—the point mass magnitudes. The point mass adjustment allows a relatively small number of parameters to express local details of the gravity field which would require perhaps a hundredfold increase in the number of parameters if the adjustment was performed only on the basis of spherical harmonics.

The results are contour maps of geoid undulations and/or gravity anomalies constructed from values computed in a grid extending over the region of interest. No point masses are present outside the local region so that the geoid there essentially coincides with the global geoid. Contour maps of standard deviations (sigmas) of the adjusted and/or predicted quantities can also be constructed.

2. MATHEMATICAL BACKGROUND

In the spherical approximation, the boundary condition and Brun's formula, respectively, read

1. Reilly, J.P., and Herbrechtsmeier, E.H. (1978) A systematic approach to modeling the geopotential with point mass anomalies, J. of Geophys. Res. 83(No. B2).

$$\Delta g = (\partial T/\partial r)_{r=R} - (2/R)\,T, \tag{1}$$

$$N = T/G\,, \tag{2}$$

where

Δg = gravity anomaly,

N = geoid undulation,

T = disturbing potential,

R = earth's mean radius (6371 km),

G = average value of gravity (980 gals).

These two formulas can be found, for example, on pages 88 and 94 of Heiskanen and Moritz.[2]

Although the above two formulas usually refer to the normal, or ellipsoidal, field, in this report they indicate a higher order field, specifically the spherical harmonic (S. H.) model after the first adjustment. Thus, T will depict the disturbing potential associated with point masses (P. M.) and will describe the field beyond the S. H. model (for example, beyond the 14, 14 model). Accordingly, the quantities Δg, N, and T appearing in Eqs. (1) and (2) could be written with the subscript "tot" if they refer to the normal field, and with the subscript "P. M." if they refer to a higher order field represented by the S. H. model. Since it holds by definition that

$$W = U + T_{tot}\,,$$

where

W = actual potential,

U = normal potential,

T_{tot} = (total) disturbing potential,

and since we have chosen to separate the total disturbing potential into two parts,

$$T_{tot} = T_{S.H.} + T_{P.M.}\,, \tag{3}$$

2. Heiskanen, W. A., and Moritz, H. (1967) Physical Geodesy, W. H. Freeman and Co., San Francisco, CA.

it follows from Eqs. (1) and (2) that

$$\Delta g_{tot} = \Delta g_{S.H.} + \Delta g_{P.M.} , \tag{4}$$

$$N_{tot} = N_{S.H.} + N_{P.M.} . \tag{5}$$

Similar relations will be used in actual adjustments, not only for predictions but also for their variances.

The parts denoted "S. H." can be treated either in the spherical approximation or in a more accurate form. On the other hand, the "P. M." parts can always be treated in the spherical approximation; after the first, S. H. adjustment, the residuals to be accommodated by the subsequent P. M. adjustment are usually at least one or two orders of magnitude smaller than the original observations (for example, geoid undulations) so that this kind of approximation could not appreciably contaminate the resulting geoid representation, gravity anomalies, and so on. By contrast, if the "total" quantities in Eqs. (4) and (5) were represented by a model containing the spherical approximation, the error thus introduced could amount to several decimeters in geoidal heights. It is thus clear that if the P. M. model containing the spherical approximation were used in an adjustment of the "total" quantities referring directly to the normal field, a decimeter accuracy in the geoid determination could not even be approached, due to this approximation alone.

Since we shall henceforth deal only with the "P. M." part of the quantities in Eqs. (3) through (5), and so on, the subscript "P. M." can be safely omitted. Based on the relation

$$T_i = \sum_j (1/\ell_{ij}) (kM)_j , \tag{6}$$

where

ℓ_{ij} = distance between the i-th (observation) point and the j-th point mass,

$(kM)_j$ = j-th scaled point mass,

from Eqs. (1) and (2) we can derive

$$\Delta g_i = (1/R) \sum_j (1/\ell_{ij}) [(R^2 - F_{ij})/\ell_{ij}^2 - 2] (kM)_j , \tag{7}$$

$$N_i = (1/G) \sum_j (1/\ell_{ij}) (kM)_j , \quad (8)$$

where

$$\ell_{ij} = (R^2 + R_1^2 - 2F_{ij})^{1/2} ,$$

$$R_1 = R - d ,$$

$$F_{ij} = RR_1 \cos \psi_{ij} ,$$

$$\cos \psi_{ij} = \sin \phi_i \sin \phi_j + \cos \phi_i \cos \phi_j \cos (\lambda_i - \lambda_j) .$$

The spherical approximation becomes further apparent from the fact that the point masses are considered to be located on a sphere of radius R_1, at the depth d beneath the surface of the sphere approximating the earth.

These equations are adapted from formulas which appeared in Needham[3] and Blaha[4]. They also agree with the algorithm presented in Reilly[1]. In the latter reference, however, the formulas referred to the ellipsoid and not to a higher order surface; with the exception of normal gravity, they were based entirely on the spherical approximation.

3. RESULTS OF ALTIMETER DATA REDUCTIONS

The data in the first global adjustment performed at AFGL consist of the available GEOS-3 satellite altimeter observations and the global set of 1654 equal 5° mean gravity anomalies supplied by R. Rapp, Ohio State University. The gravity anomaly data is based upon approximately 38,000 1° mean anomalies. Accuracy estimates for the 5° mean anomalies are depicted in Figure 1. Of the 1654 5° mean anomalies 1507 were based directly on the 1° anomalies, while the remaining 147 5° means were obtained by interpolation. The mean gravity anomalies serve mainly to make the spherical harmonics adjustment possible. Because of data gaps over the continental regions, the adjustment could not be performed with the use of satellite altimetry alone.

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3. Needham, P.E. (1970) The Formation and Evaluation of Detailed Geopotential Models Based on Point Masses, Department of Geodetic Science, Report No. 149, The Ohio State University, Columbus, Ohio.

4. Blaha, G. (1977) Refinements in the Combined Adjustment of Satellite Altimetry and Gravity Anomaly Data, Report of DBA Systems, Inc.; AFGL-TR-77-0169, AD A047597.

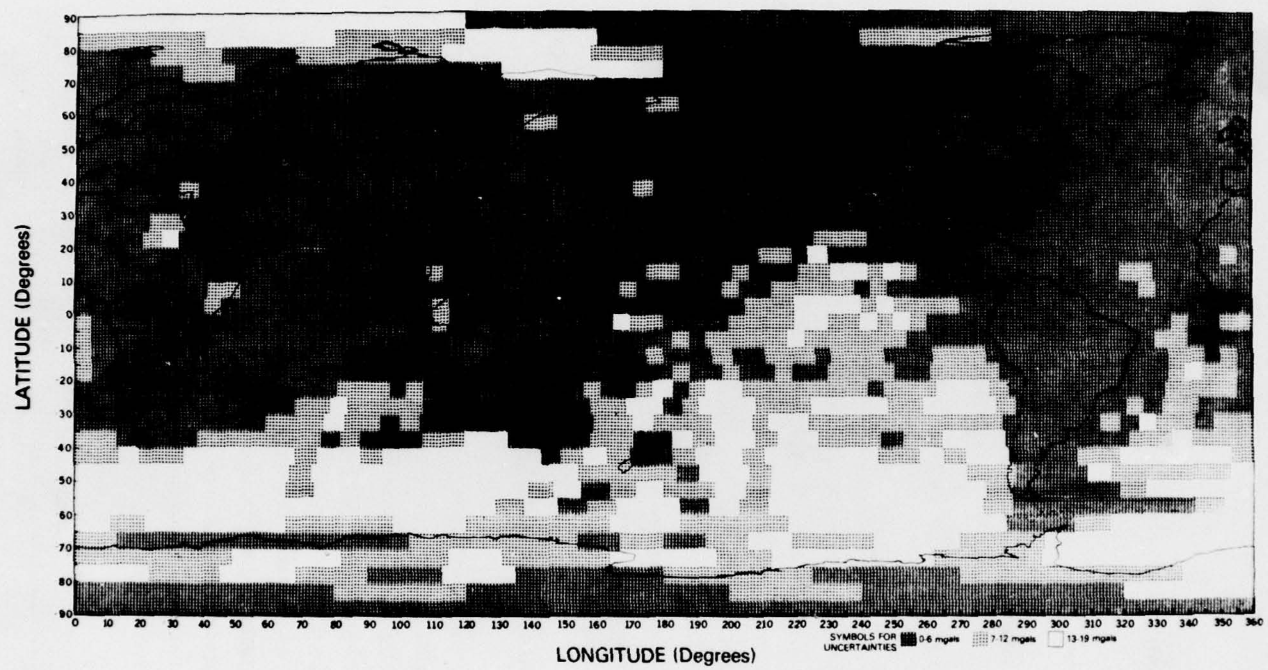

Figure 1. Five-Degree Equal Area Anomalies and Their Uncertainties

Figure 2 depicts the ground tracks of 1253 GEOS-3 passes, over oceanic regions, used in this global adjustment of satellite altimetry and gravity anomalies. The spherical harmonic model is complete through the degree and order (14, 14). The altimeter data collected over the North Atlantic region are further used in a subsequent point mass adjustment.

In the second or residual adjustment, the input provided by satellite altimetry completely overrides any influence by the gravity anomaly input because over the region of interest, such as the North Atlantic, the altimeter data density is hundreds or thousands of times higher than the density of data coming from the anomalies and the weight associated with each altimeter observation is considerably higher than the weight associated with any given gravity anomaly. Accordingly, the parameters in the second adjustment providing the local geoidal detail are the point mass magnitudes, and the "observations" in the area of concern are the minus residuals of geoid undulations as obtained from satellite altimetry in the first adjustment; the input weights are unchanged from those in the first adjustment.

The results from the first adjustment are shown in Figures 3 and 4. Figure 3 depicts contours of geoid undulations in 10-m intervals and Figure 4 depicts contours of gravity anomalies in 10-mgal intervals, both on the global scale over the oceanic regions. The reference ellipsoid used in these computations has a flattening of 1/298.24 with a semi-major axis of 6,378,160 m (Geodetic Reference System, 1967).

The point mass locations are represented by dots in Figure 5. Their horizontal separation is 2° in latitude and longitude, forming almost equilateral blocks near the equator. A 2° separation is suitable for representing geoidal detail corresponding to a half wavelength of about 4°, that is, two parameters per half wavelength in each direction. The depth of the point masses corresponds to approximately 1.6 times their horizontal separation, in this case about 350,000 m. This configuration has been confirmed through computer simulations, and seems to be a suitable compromise resulting in a sufficiently accurate description of the gravity field, without exceedingly large correlations among the adjusted parameters.

Figure 5 also shows contours of geoid undulations in 5-m intervals obtained when the point mass adjustment is superimposed, in the area of interest, on the previous spherical harmonic adjustment. One can detect higher geoidal resolution within the area covered by the point masses. Remarkable similarities in the shape of the local geoid have been observed between this geoidal map and that produced in the North Atlantic calibration area by Marsh and Chang in 1976, who based their geoid on a combination of 5 ft × 5 ft, 15 ft × 15 ft, and 1° × 1° surface gravity data and the GSFC GM-8 Earth Model.

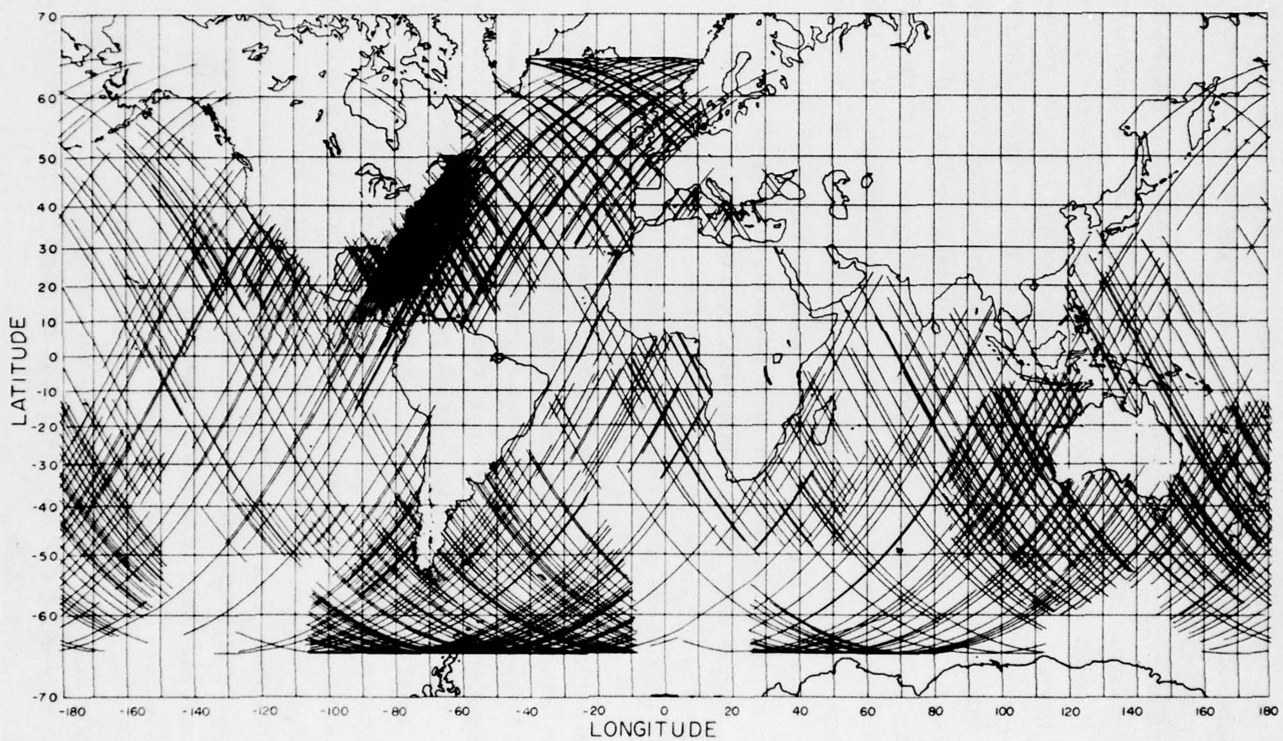

Figure 2. Ground Tracks of 1253 GEO3-3 Passes

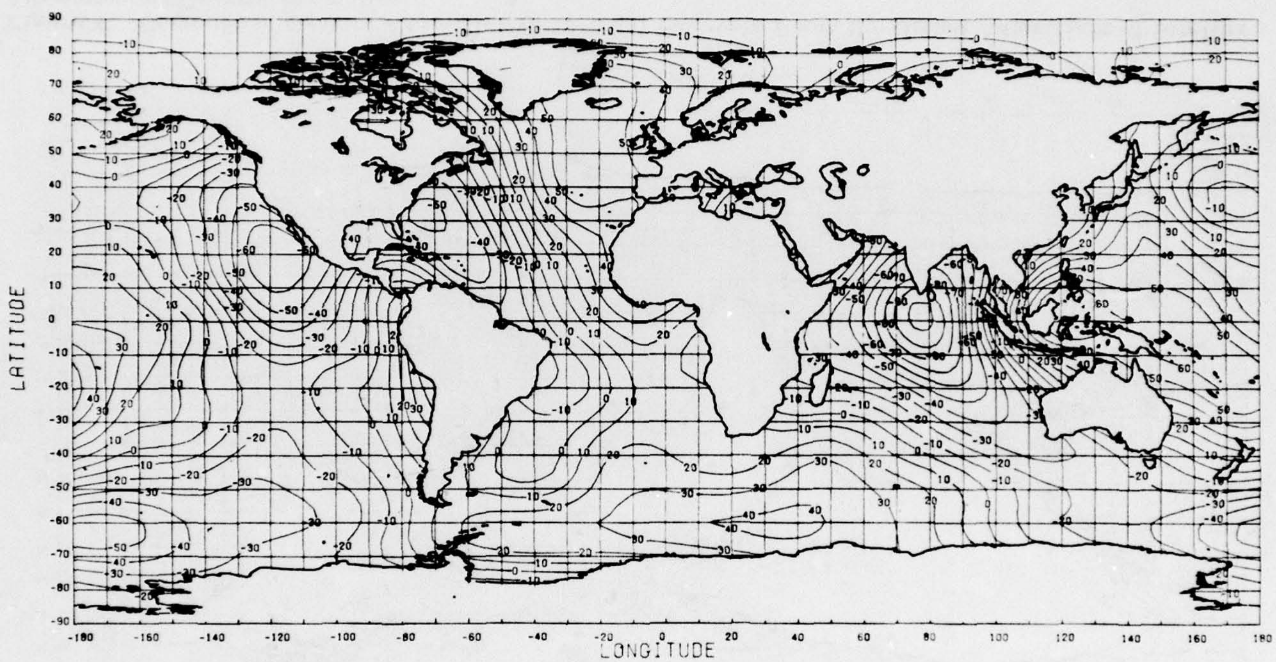

Figure 3. Contours of Geoid Undulations in 10-m Intervals From the Global Adjustment of Satellite Altimetry and Gravity Anomalies

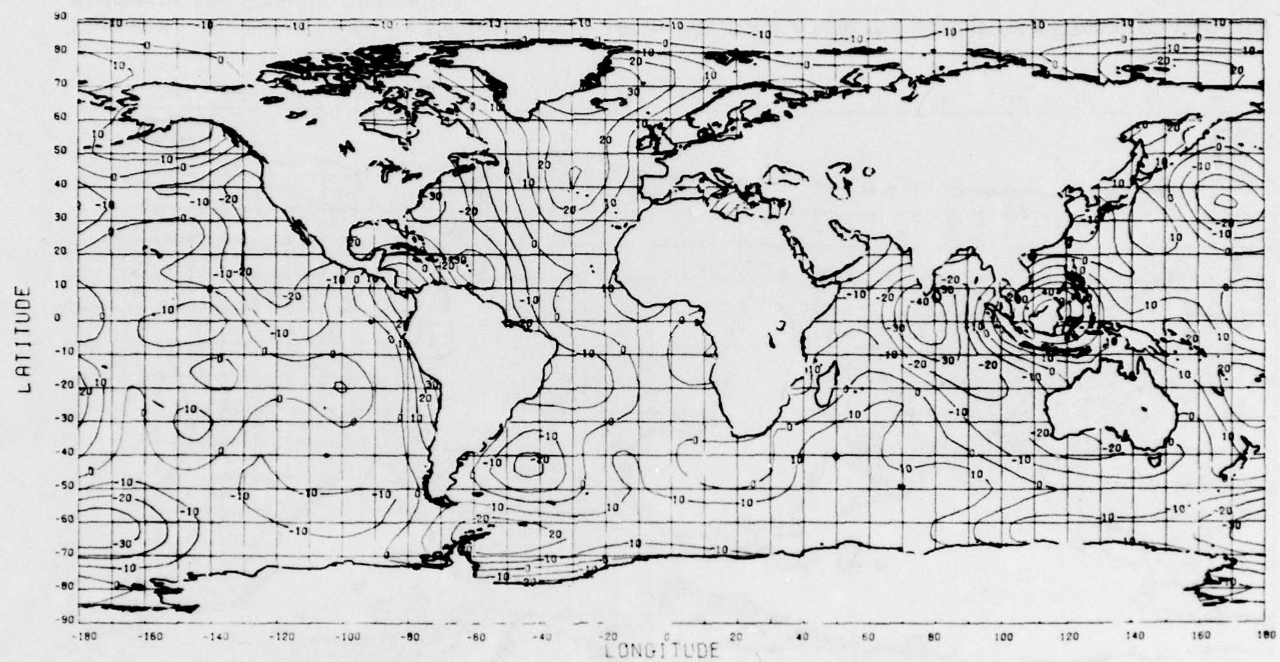

Figure 4. Contours of Gravity Anomalies in 10-mgal Intervals From the Global Adjustment of Satellite Altimetry and Gravity Anomalies

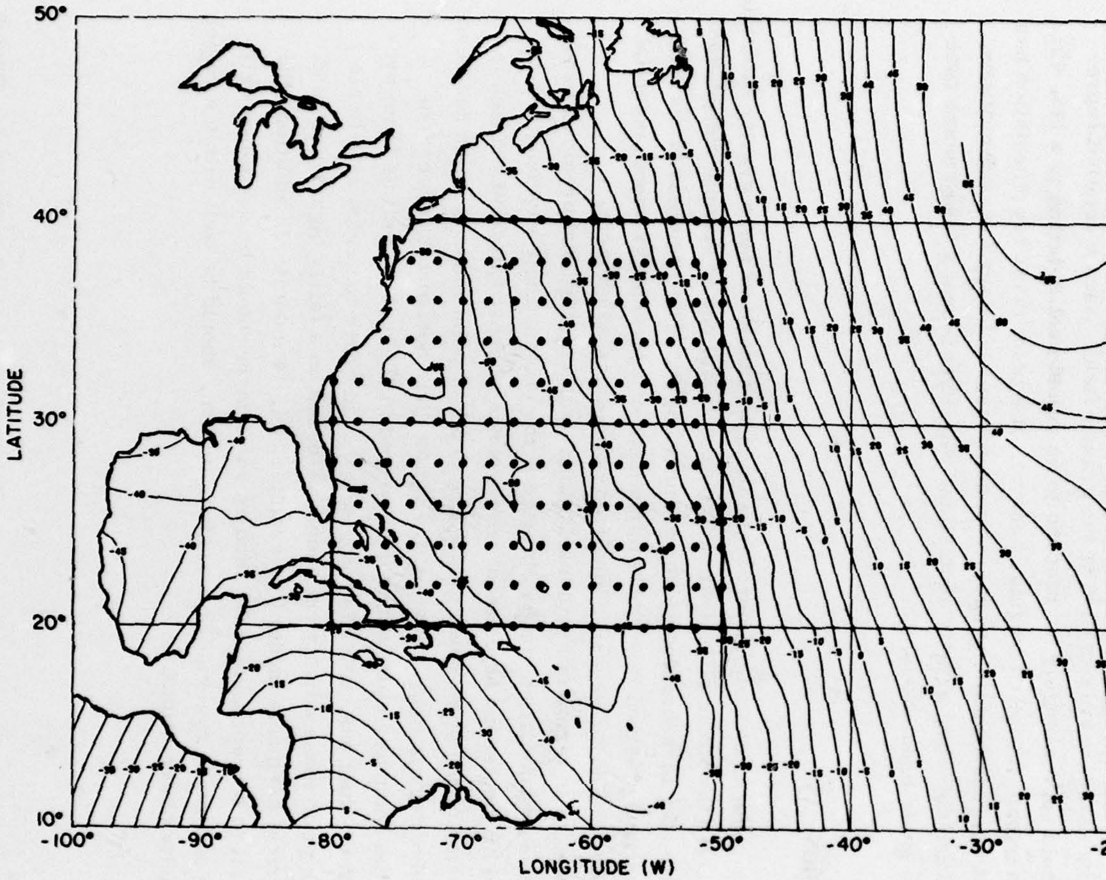

Figure 5. Geoid Undulation Contours From a Spherical Harmonic 14-Degree Solution and a Point Mass Adjustment Superimposed on the Global Adjustment Using Combinations of Gravity Anomaly and Altimetric Data

Finally, Figure 6 depicts the predictions of free air gravity from the same adjustment. The extreme left- and right-hand portion of this figure shows the smooth contour lines based essentially on the global adjustment in terms of a (14, 14) spherical harmonic model, while the remaining portion shows a much more detailed (point mass) representation of the gravity field. Such a detailed representation would approximately correspond to an adjustment in terms of a (45, 45) spherical harmonic model. Irregular behavior of a point mass representation has been noticed in areas containing several point masses but very few observations, especially when depicting gravity anomalies, which represent a less smooth function than the geoid undulations.

4. CONCLUSION

In many present-day adjustment tasks the data consist of mean gravity anomalies distributed over the whole globe (with some exceptions) and of densely distributed satellite altimeter measurements over a limited area. The residuals of the altimeter observations obtained in a global adjustment of spherical harmonic potential coefficients enter the subsequent, localized point mass adjustment.

Good results in terms of geoid undulations (both adjusted and predicted) and predicted gravity anomalies can be obtained if the shortest half wavelength to be represented by the second adjustment corresponds to about two point masses in each dimension of the point mass grid. The depth of point masses which has been used with advantage is approximately 1.6 times their horizontal separation.

From the computer simulations documented in Blaha,[5] as well as from preliminary data reductions, it further appears that in order to arrive at reliable results, the point mass adjustment should be based on a fairly high degree and order spherical harmonic model (for example, 14, 14 model). In addition, the geoidal detail to be represented by such an adjustment should be contained well within the point mass region. This region, in turn, should be well within a region of high density observations.

5. Blaha, G. (1979) Improved Determination of the Earth's Gravity Field, Report of Nova University, AFGL-TR-79-0058.

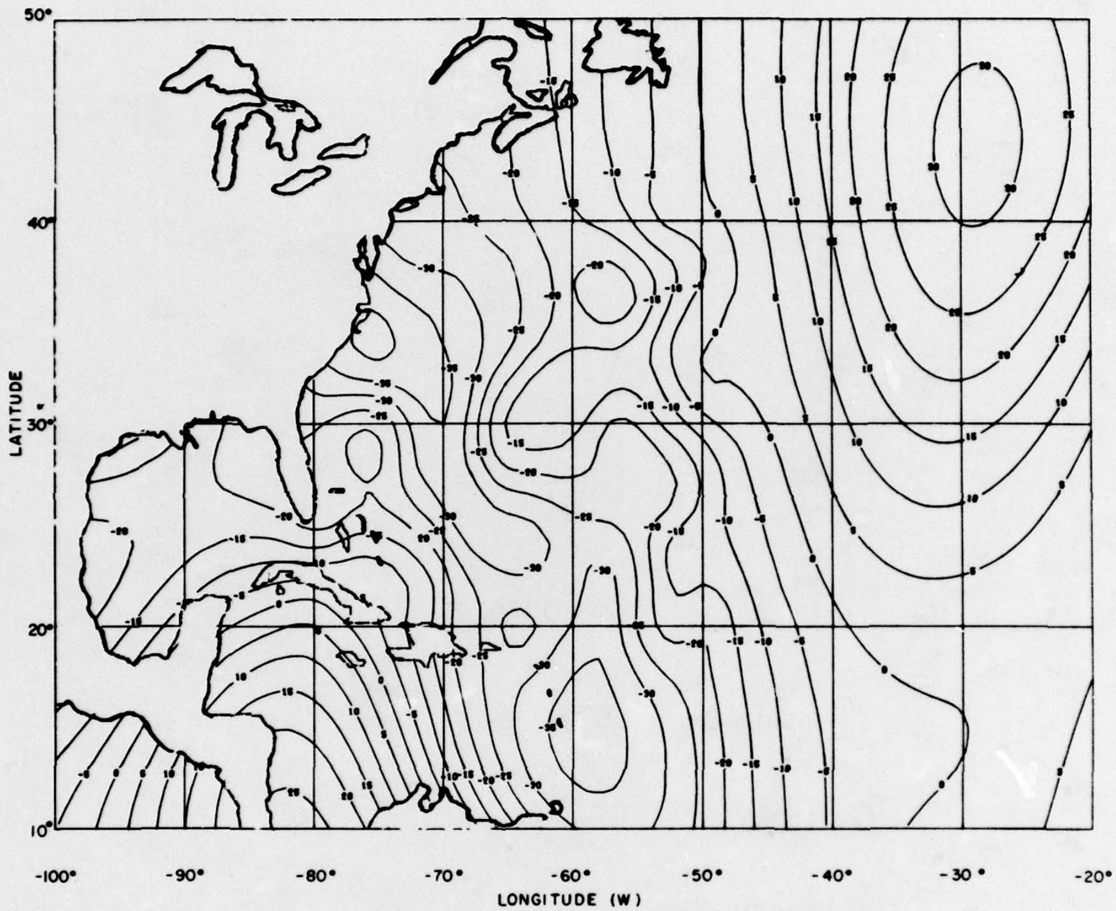

Figure 6. Free Air Gravity Anomaly Contours From a Spherical Harmonic 14-Degree Solution and a Point Mass Adjustment Superimposed on the Global Adjustment Using Combinations of Gravity Anomaly and Altimetric Data